

# **An Optimal Investment Strategy with Multivariate Jump Diffusion Models •**

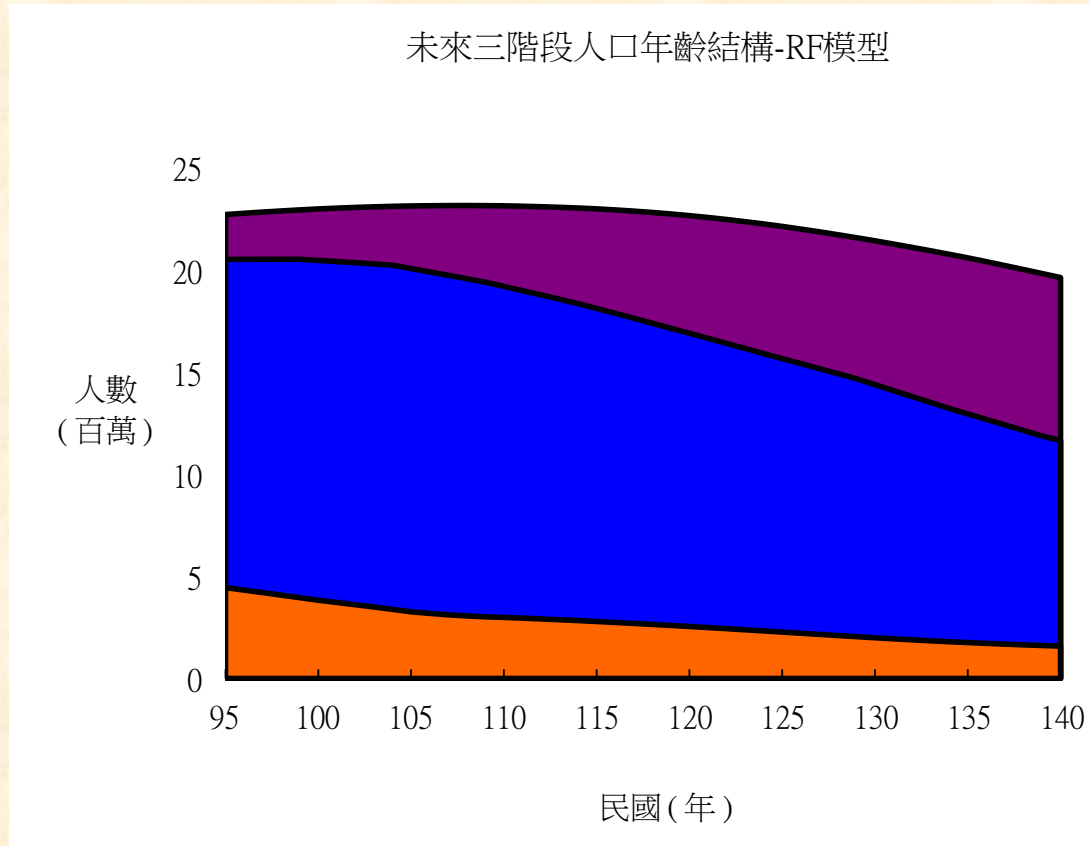
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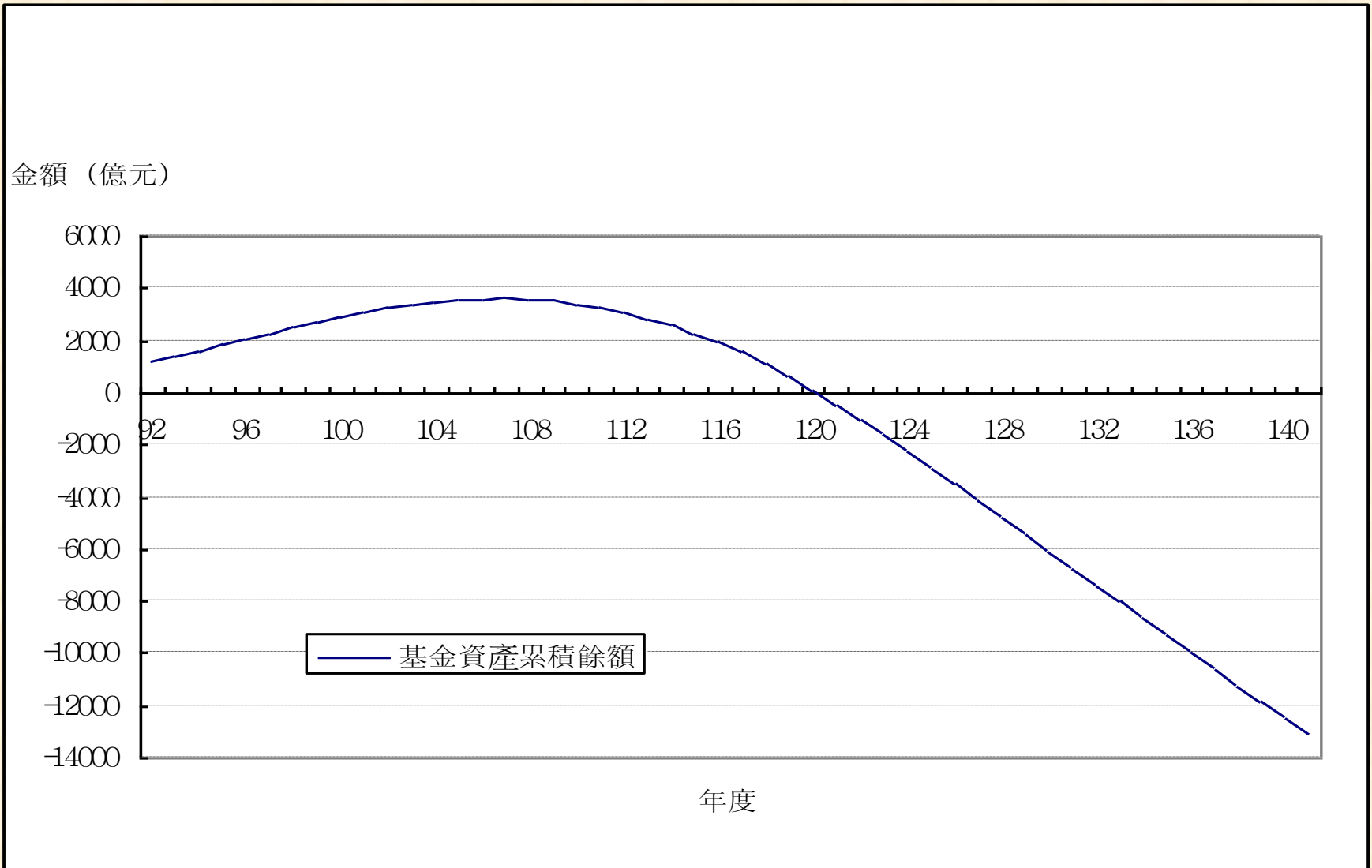
# Background

- Financial problems of pension funds are serious.
  - Increase contribution rate
  - Decrease retirement benefit
  - **Enhance investment return of pension fund**

# Prediction of Aging Population



# Prediction of Pension Fund



# Purpose

- **Construct a good asset model**—a multivariate jump diffusion (MJD) model.
- **Choose objective functions** for fund management.
- The proposed MJD model **provides more detailed information** about the financial crisis.
- Allow fund managers to **determine an appropriate asset allocation strategy** that enhances investment performance during the crisis.

# Multivariate Lévy processes

- One way to incorporate **both kurtosis and skewness** for the probability distribution of assets returns is **multivariate Lévy processes**.

# The first goal of this paper

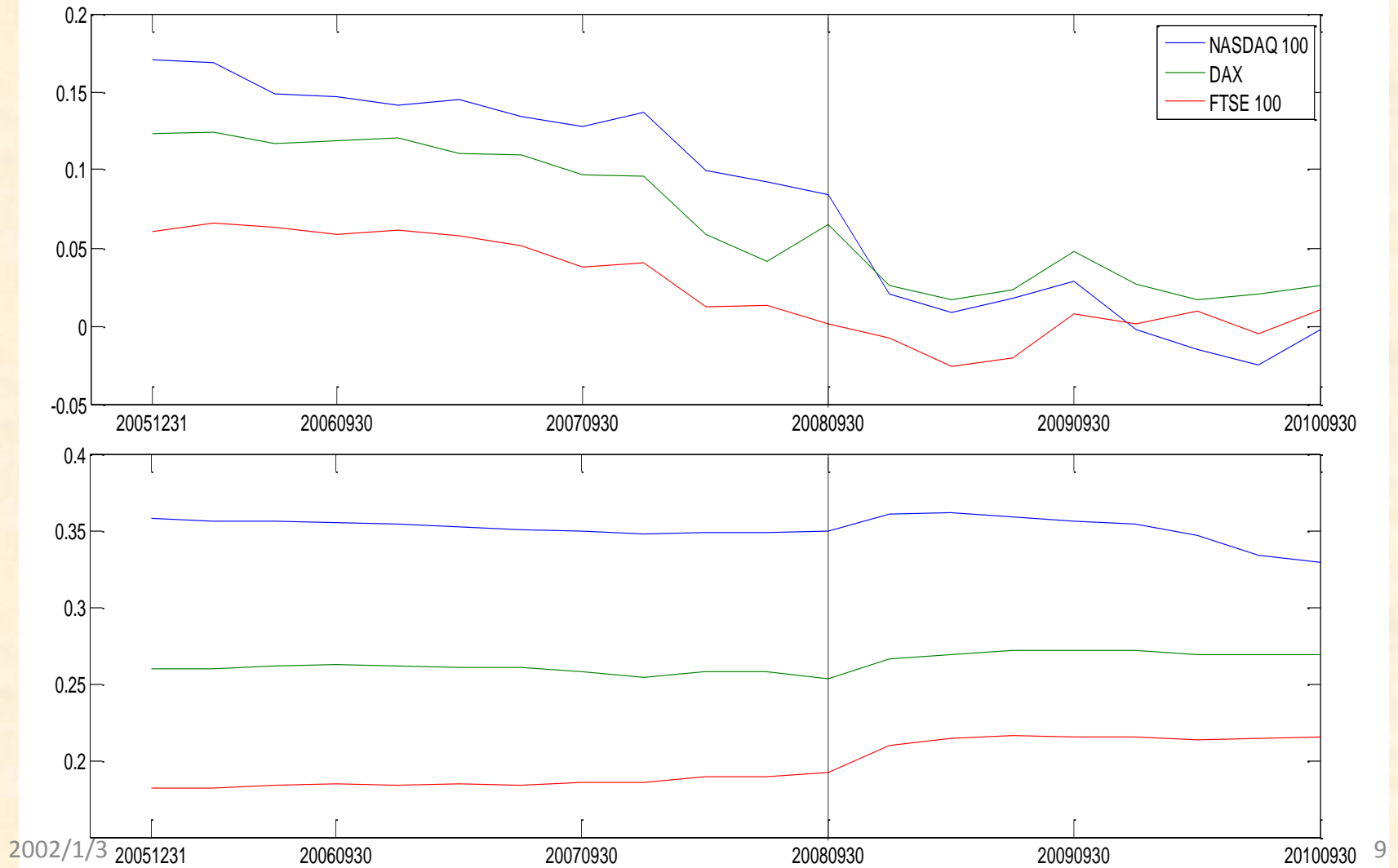
- Luciano and Schoutens (2010) provide a multivariate time-changing model that incorporates both a time transform common to all assets and an idiosyncratic one for multivariate compound Poisson processes **with zero-mean jump size**.
- In this article, different from Luciano and Semeraro (2010), we construct generalized multivariate compound Poisson processes **with nonzero-mean jump size and multivariate jump diffusion processes that integrate idiosyncratic and systematic jumps simultaneously**.

# Fund Management

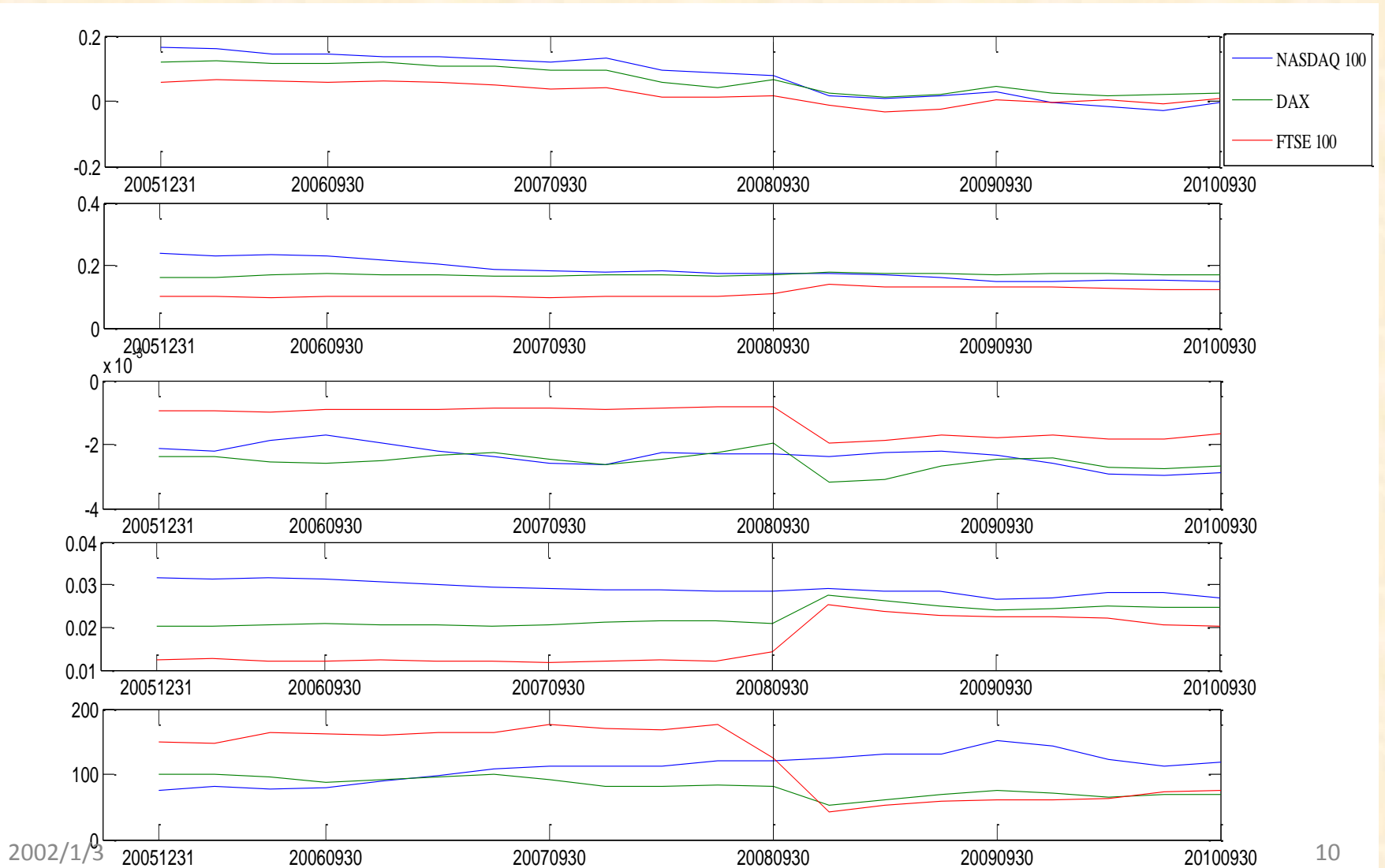
- Assume a fund invests in three stock indices: **NASDAQ100, DAX and FTSE100.**
- Choose a feasible investment strategy that **minimizes the discounted future cost.**
- Derive **a first approximation of the optimal multiperiod asset allocation** at the beginning of the term.
- **Reassess** the optimal investment strategy **at every decision date**, which then **takes all new information** into account at each decision date
- **Revises** the optimal asset allocation **for the rest of the periods until maturity.**



# Calibrated parameters of MN model



# Calibrated parameters of MJD model



# Objective Function

$$\textcircled{1} V(\mathfrak{S}_t) = \min_{\{\pi_t\}} E(G(t) | \mathfrak{S}_t)$$

where  $\{\pi_t\} = \{w(s); s = t, t + 1, \dots, n - 1\}$

represents the set of the future possible investment strategies

$$\textcircled{2} G(t) = \sum_{u=t}^n v^{s-t} C(u)$$

$$\textcircled{3} C(t) = \theta_t (F^*(t) - F(t))^2 + \kappa (F^*(t) - F(t)),$$

for  $t = 1, \dots, n,$

where  $\theta_t = 1, \text{ for } t = 1, \dots, n - 1$  and  $\theta_n = \theta$

# Parameter Calibration

- With data from **January 1, 1996, to December 31, 2010**, we empirically test the stock indices using the MN and MJD models. In the empirical results.
- The calibrated parameters rely on a **rolling window** sample procedure, **fixed at ten years**.
- The **first** rolling sample included stock index returns from **January 1, 1996, to December 31, 2005**.
- Both the MN and MJD models are re-estimated every quarter to update the distribution parameters of each model.
- The estimated sample can be rolled forward by **omitting the returns for the oldest quarter** and **adding the returns for the latest quarter**.
- This procedure repeats until the **final sample**, which ranges from **October 1, 2001, to September 30, 2010**.
- Therefore, there are 20 parameter sets for the MN and MJD models.

# Simulation Procedure for Optimal Portfolio Asset Allocation

- Step 1: With terminal data of the  $j^{\text{th}}$  time period, for  $j = 1, \dots, n$ , and using the stock indices in the terminal data as the initial stock indices, **we generate the additional  $21 - j$  stock prices as the stock indices in adjacent quarters for each simulated path**, using the calibrated parameters of the MN and MJD models. For this research, we generate **10,000 simulated paths**.

# Simulation Procedure for Optimal Portfolio Asset Allocation

- Step 2: With the simulated paths, we obtain the optimal multiperiod asset allocations for each quarter by minimizing the objective function.

# Simulation Procedure for Optimal Portfolio Asset Allocation

- Step 3: We choose **the weights for the first quarter** in the optimal multiperiod asset allocations as the optimal weights in the  $j^{\text{th}}$  time period for  $j = 1, \dots, n$ . Then we take all new information into account in each quarter and **revise the optimal asset allocation for the rest of the periods at each quarter until maturity.**

# Comparing Two Stochastic Investment Models

- ① Multi-Asset Model: Multivariate Normal Process
  
- ② Multi-Asset Model: Multivariate Jump Diffusion Processes



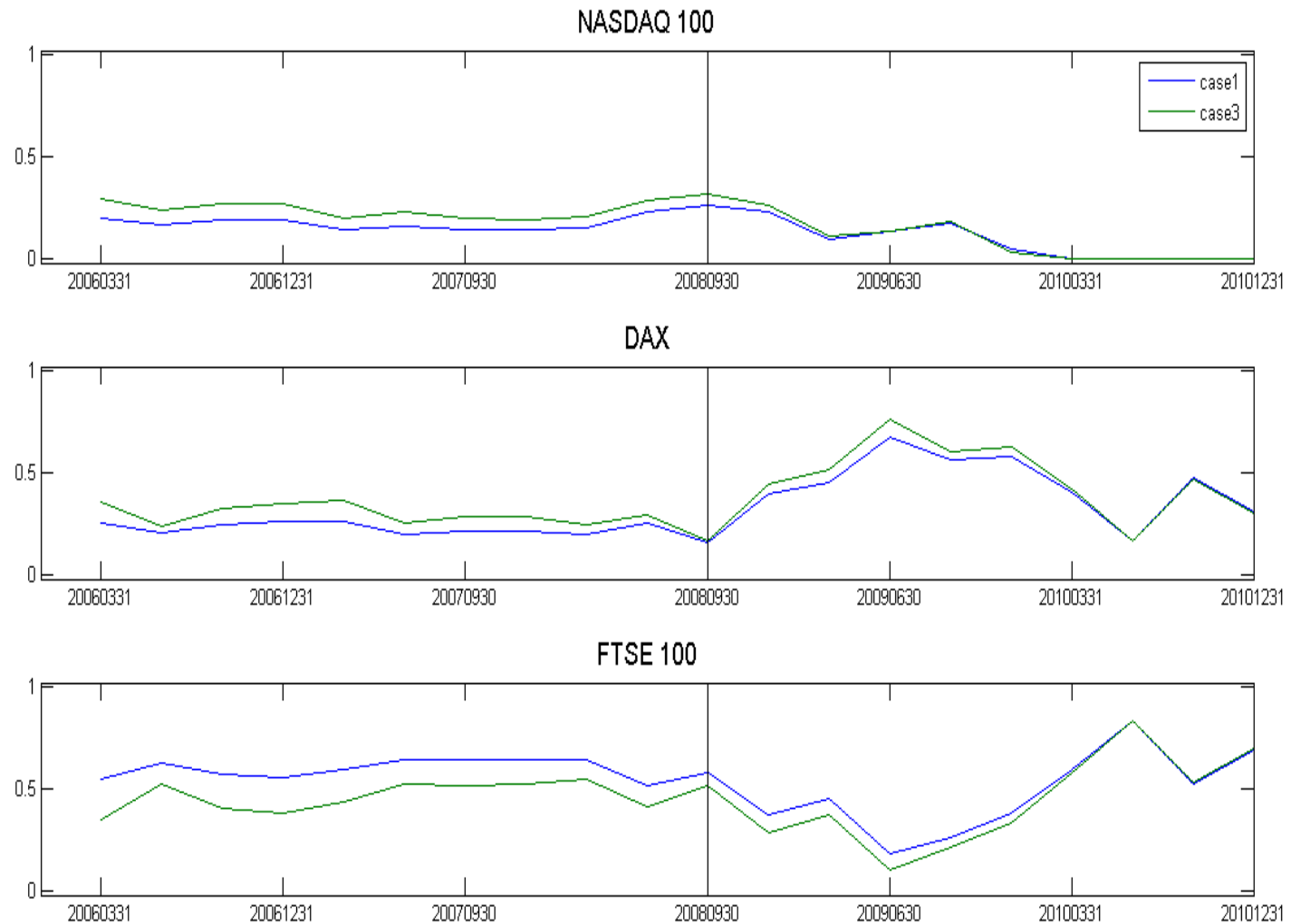
# Different cases of asset model and objective strategies

	Asset Model	$\kappa$	$\theta$
Case 1	MJD	1	1
Case 2	MJD	1000	1
Case 3	MJD	1	1000
Case 4	MN	1	1
Case 5	MN	1000	1
Case 6	MN	1	1000

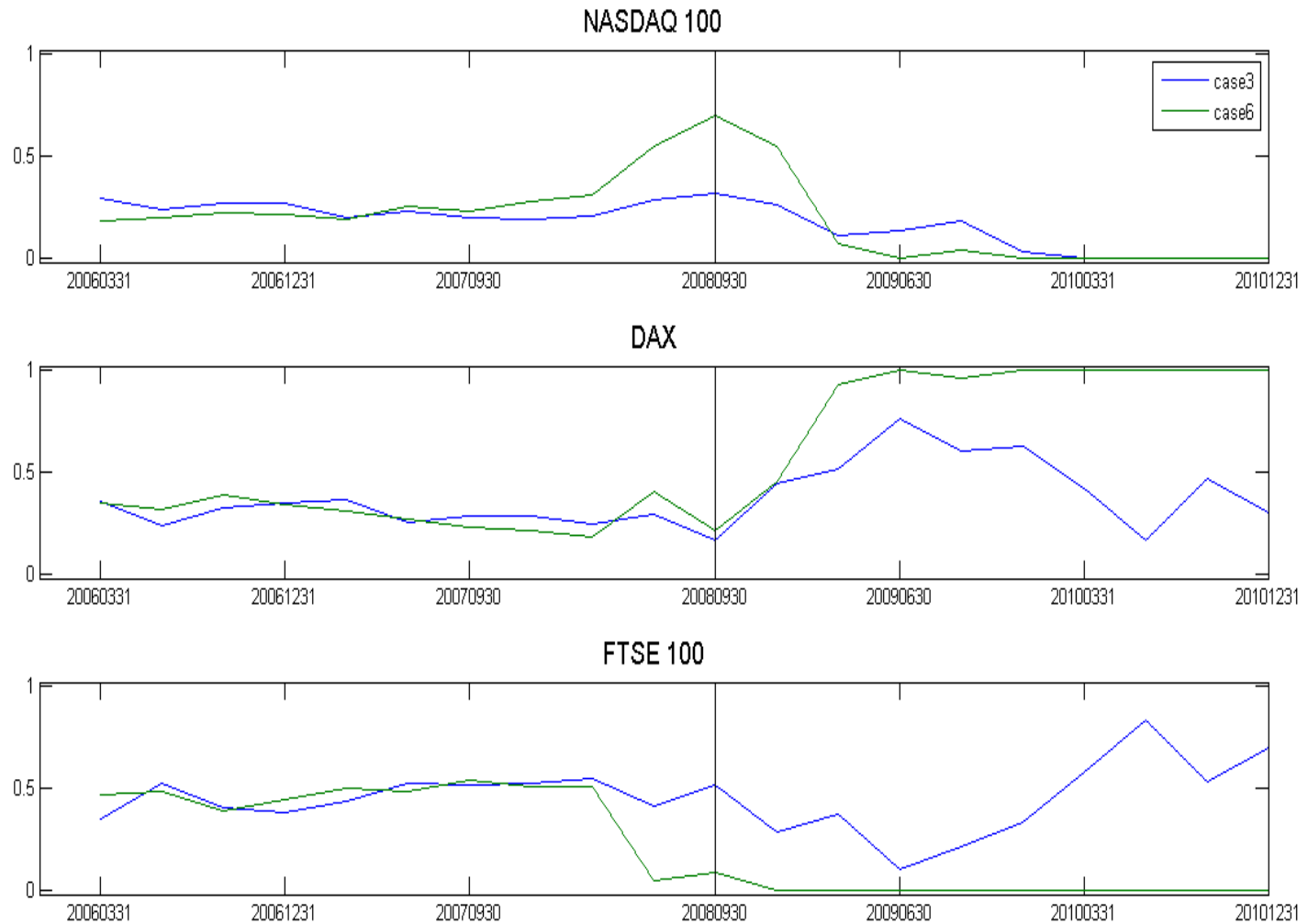
# The real fund values of the out-of-sample performance for these six cases and the target fund values during the 20 quarters

Data	20060331	20060930	20070331	20070930	20080331	20080930	20090331	20090930	20100331	20100930
Fund Value (Case 1)	105.39	103.52	111.16	118.94	103.79	89.16	63.66	85.20	93.91	91.06
Fund Value (Case 2)	101.81	98.14	107.91	123.78	107.08	90.76	58.19	78.36	85.51	87.00
Fund Value (Case 3)	105.48	103.72	112.67	122.20	106.33	91.29	64.59	86.9	95.72	92.91
Fund Value (Case 4)	105.44	103.81	111.27	119.44	104.39	88.84	59.04	79.52	86.77	88.28
Fund Value (Case 5)	101.41	97.60	105.79	123.48	106.84	90.56	57.14	76.95	83.97	85.43
Fund Value (Case 6)	105.86	104.19	112.67	122.79	106.92	91.05	60.29	81.20	88.61	90.15

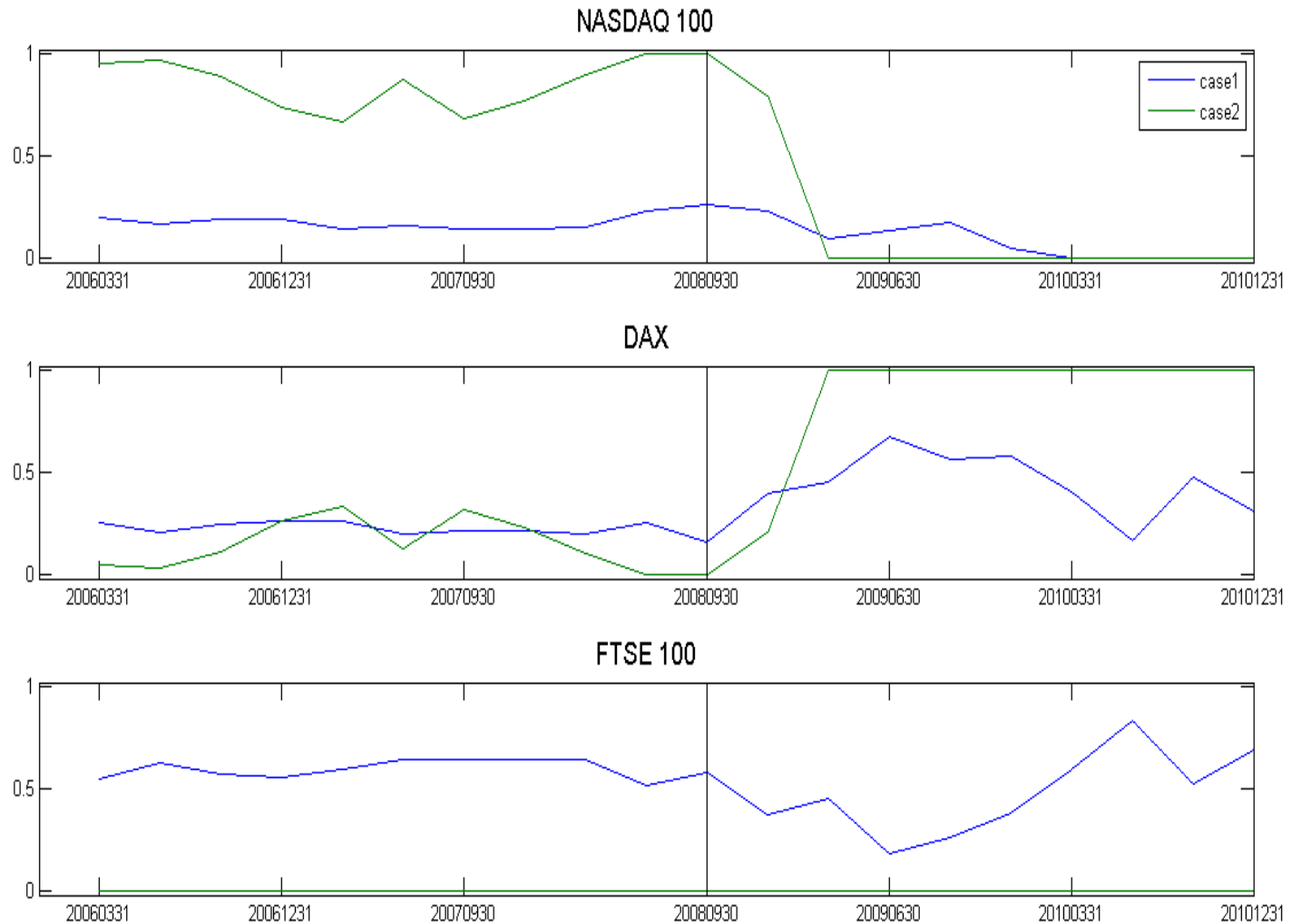
# Optimal proportion : Case 1 vs Case 3 (terminal matching)



# Optimal proportion : Case 3 vs Case 6



# Optimal proportion : Case 1 vs Case 2(downside risk)



# Conclusions

- Investment Strategy
- Asset Models
- Objective Functions
- Asset Selections

# Thanks